Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol.131, No.10, pp.1243-1251, 2005. 10.

Fixed-Head Pile Bending by Kinematic Interaction and Criteria for its Minimization at Optimal Pile Radius

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CE Database Keywords: Soil-structure interaction, Elastic foundations, Soil-pile interaction, Dynamic analysis, Embedded foundations, Seismic design

ABSTRACT: This study focuses on how to determine an optimal radius minimizing the bending strains of vertical, cylindrical fixed-head piles embedded in a homogeneous elastic stratum in soil-pile-structure systems where the kinematic interaction dominates. In order to determine the appropriate radius, closed form formulae for the bending strains at the head of the piles are derived based on three-dimensional wave propagation theory. A general expression of the closed form formulae can be obtained by normalizing the bending strains with respect to a mean shear strain of the soil medium. The normalized bending strains can be expressed by the radius to height ratio of the piles, the ratio of soil to pile stiffness, and a factor representing the relative amplitude and the phase lag between the loading at the head of the piles and the deformation of the ground. The results of parametric analyses reveal the presence of an optimal radius that locally minimizes the bending strains of the piles. Criteria for determining the optimal radius of soil-pile-structure systems are presented.

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Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol.131, No.10, pp.1243-1251, 2005. 10.

INTRODUCTION

Kinematic bending induced by the lateral deformation of soft soil surrounding a pile has become an important problem in geotechnical engineering because the kinematic bending may not be negligible when compared with the inertial bending induced by the inertial forces of a superstructure and a footing [Mizuno, et al. (1984), Ohira et al. (1985), Mizuno (1987), Tazoh et al. (1988), Kavvadas and Gazetas (1993), Kaynia and Mahzooni (1996), Mylonakis et al. (1997), Nikolaou et al. (2001), and Luo et al. (2002)]. Apparently, the kinematic bending and the inertial bending occur more or less at any resonant period in soil-pile-structure systems. In seismic design, therefore, the kinematic bending and inertial bending must be considered simultaneously when evaluating the performance of a pile. Various types of analytical and numerical methods have been proposed for estimating the dynamic response of soil-pile-structure systems in recent years [e.g., Kagawa and Kraft (1981), Takemiya and Yamada (1981), Wolf and Von Arx (1982), Kaynia (1982), Gazetas (1984), Tazoh et al. (1988), Kavvadas and Gazetas (1993), and Mylonakis et al. (1997)]. At present, therefore, the performance of a pile can be adequately evaluated by using these methods.

Mylonakis et al (1997) indicate that the inertial bending would be significant, especially at the upper part of piles, when inertial responses with the fundamental period of soil-pile- structure systems are dominant (referred to in this study as the dominance of inertial interaction). On the one hand, the kinematic bending would be significant when soil motions with the natural periods of soil strata dominate (referred to in this study as the dominance of kinematic interaction). It was found in previous analytical and field studies that damage due to the kinematic bending may occur, especially at the following three parts along a fixed-head pile: (a) the head of the pile; (b) interfaces of soil layers with sharply different shear modulus or shear strength; and (c) the toe of the pile. In general, the damage due to the kinematic bending is dominant at the head of the pile in homogeneous strata, while in the case of layered soil strata, damage to the pile is caused at the interfaces of soil layers with strong discontinuities in stiffness and strength in the soil profile, as well as at the head of the pile. Nikolaou et al. (2001) show that the kinematic bending strains at the interfaces of soil layers sometimes surpass the bending strains at the head of the pile, depending on the ratio of the stiffness of the soil layers, the pile-soil stiffness contrast, the relative depth from the head of the pile down to the interface of the layers with respect to the length of the pile, etc. The kinematic bending strains at the toe of the pile may also dominate in cases of both homogeneous and layered soil strata when the toe is strongly restrained [e.g. Ohira et al. (1985)].

In seismic design, therefore, the properties of the pile should be determined in consideration of both the inertial bending and the kinematic bending, simultaneously. In particular, it is important to determine the pile radius because the size of the radius directly affects the

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bending stiffness (flexibility) of the pile EI , which consequently affects the seismic performance of the pile. In general, when an estimated bending strain exceeds its allowed limits, engineers may use various techniques for setting the radius, based on the concept that increasing the pile radius is an appropriate solution for decreasing the bending strains. This idea is obviously based on the assumption that the loading at the head of the pile strongly affects the response of the piles, while the deformation of the soil stratum is negligible. Therefore, such techniques would be applicable for cases where the inertial interaction dominates in soil-pile-structure systems. Moreover, it is expected that, in many cases, such techniques may find an optimal radius for the pile since the largest bending strain at the head of the pile may generally be controlled by the inertial interaction. To some extent, however, kinematic bending could also control the bending strain if the kinematic interaction is predominant. In fact, there have been few investigations into pile bending characteristics in systems where the kinematic interaction dominates. In practical engineering, therefore, no specific techniques for determining the optimal radius of a kinematically affected pile are available, even for the simplest case of a homogeneous soil profile.

Accordingly, this study focuses on the bending strains at the head of vertical, cylindrical fixed-head piles embedded in a homogeneous elastic stratum, and the effect of the pile radius on the bending strains, in soil-pile-structure systems where the kinematic interaction dominates. Supported by the results of previous studies [e.g., Nikolaou et al. (2001)], the specific period particularly important in this study is the fundamental natural period of the homogeneous soil stratum. Analytical results evaluated by applying the Beam-on-Dynamic-Winkler-Foundation (BDWF) method indicate that the maximum values of the kinematic bending strains at the head of the piles occur at the fundamental natural period of soil strata for most soil profiles [Nikolaou et al. (2001)]. The above results imply that it would be worth investigating the behavior of the bending strains at the fundamental natural period of the soil stratum to understand their fundamental characteristics.

The objectives of the present study are: (1) To obtain the fundamental relation between the radius and the bending strains at the head of the piles using three-dimensional wave propagation theory; (2) to show the presence of an optimal pile radius that minimizes the bending strains at the head of the piles; and (3) to derive criteria by which the optimal radius can be determined for soil-pile-structure systems.

SYSTEM STUDIED

The soil-pile-structure system is shown in Fig. 1. A vertical, cylindrical pile of radius a is embedded in a homogeneous elastic stratum of thickness H . The toe of the pile is supported by a compliant bedrock. A cylindrical coordinate system, r, θ, z , is considered, with the origin taken at the center of the base of the pile. Complete bonding at the interfaces between the pile and soil medium is assumed. The toe of the pile is presumed to be restrained elastically against rotational movements by a spring of static stiffness K_r at the base. In this study, the stiffness K_r assumed is described by the following formula [Veletsos and Wei (1971)]:

$$K_r = \frac{8a^3 \rho_b V_{sb}^2}{3(1-\nu_b)}, \tag{1}$$

where ρ_b, V_{sb}, and ν_b are the density, the shear velocity, and the Poisson's ratio of the compliant bedrock, respectively.

The material damping for the soil medium h_g is of a frequency-independent hysteretic type [Nogami and Novak (1977)]. The horizontal excitation of the base is assumed to be a steady-state vibration $u_g e^{i\omega t}$. The frequency of the horizontal excitation is assumed to be equal to the natural frequency of the soil medium, as explained above. The lateral load acting on the head of the pile, which is generated by the dynamic response of a superstructure and a footing, is represented by V.

Theoretical models similar to the above, except for the condition of the toe of the pile, are used for deriving closed form formulae for the earthquake responses of piles [Tajimi (1969), Ohira et al. (1985)]. Based on these sophisticated known techniques, the closed form formula of the bending strains at the head of the pile can be derived for the present model.

THEORETICAL SOLUTION

The closed form formula is derived based on the theoretical model described above. The governing equations of the present model and the derivation of this formula are substantially identical to those described in Tajimi (1969) and Ohira et al. (1985). Therefore, the resultant formula is simply presented here without showing the derivation. It is found through the derivation that the bending strains at the head of the pile ε_p normalized with respect to a mean shear strain of the soil medium γ_s can be expressed in terms of the normalized parameters explained below. The mean shear strain is defined as the absolute value of the maximum harmonic response displacement of the ground surface with respect to the bedrock divided by the height of the soil medium H. The normalized parameters are: (1) the slenderness ratio (the radius to height ratio of the piles, a/H); (2) the ratio of the stiffness (Young's Modulus) of the soil E_g and the pile E_p (E_g/E_p); (3) the ratio of the stiffness of the soil E_g and the compliant bedrock E_b (E_g/E_b); and (4) the ratio of the mass density of the pile ρ_p and the soil ρ_g (ρ_p/ρ_g).

The closed form formula of the normalized bending strains can be written as follows:

$$
\begin{aligned}
\left.\frac{\varepsilon_p}{\gamma_s}\right|_{z=H} = {} & c_k\left\{A_g\left[\sin\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\alpha}_n\right]+B_g\left[\cos\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\beta}_n\right]\right. \\
& \left.+C_g\left[-\sinh\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\gamma}_n\right]+D_g\left[-\cosh\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\delta}_n\right]+\sum_{n=1,3\ldots}^{\infty} g_n\bar{\varphi}_n\right\} \\
& +\frac{1}{\gamma_s}\frac{V}{E_pH^2}c_m\left\{A_f\left[\sin\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\alpha}_n\right]+B_f\left[\cos\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\beta}_n\right]\right. \\
& \left.+C_f\left[-\sinh\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\gamma}_n\right]+D_f\left[-\cosh\bar{\lambda}+\sum_{n=1,3\ldots}^{\infty} f_n\bar{\delta}_n\right]\right\}
\end{aligned}
\tag{2}
$$

where

$$l_r=\frac{a}{H},\quad r_r=\frac{E_g}{E_b},\quad s_r=\frac{E_g}{E_p},\quad d_r=\frac{\rho_p}{\rho_g}$$

$$\bar{\lambda}^2=\frac{\pi^2}{2(1+\nu)}l_r^{-2}s_rd_r,\quad \bar{h}_n=\frac{n\pi}{2}$$

$$\xi_n^2=n^2(1+i\,2h_g)-1,\quad \gamma_{s0}=\sum_{n=1,3\ldots}^{\infty}\frac{4}{n\pi}\frac{(-1)^{\frac{n-1}{2}}}{\xi_n^2}$$

$$c_k=\frac{1}{\gamma_{s0}}l_r\bar{\lambda},\quad c_m=\frac{2}{\pi}\frac{1}{\bar{\lambda}}l_r^{-3}$$

$$\bar{\alpha}_n=\left(\frac{\bar{h}_n}{\bar{\lambda}}\right)^2\frac{2\bar{\lambda}}{\bar{h}_n^{\,2}-\bar{\lambda}^2}\cos\bar{\lambda},\quad \bar{\beta}_n=\left(\frac{\bar{h}_n}{\bar{\lambda}}\right)^2\left[-\frac{2\bar{\lambda}}{\bar{h}_n^{\,2}-\bar{\lambda}^2}\sin\bar{\lambda}+\frac{2\bar{h}}{\bar{h}_n^{\,2}-\bar{\lambda}^2}(-1)^{\frac{n-1}{2}}\right]$$

$$\bar{\gamma}_n=\left(\frac{\bar{h}_n}{\bar{\lambda}}\right)^2\frac{2\bar{\lambda}}{\bar{h}_n^{\,2}+\bar{\lambda}^2}\cosh\bar{\lambda},\quad \bar{\delta}_n=\left(\frac{\bar{h}_n}{\bar{\lambda}}\right)^2\left[\frac{2\bar{\lambda}}{\bar{h}_n^{\,2}+\bar{\lambda}^2}\sin\bar{\lambda}+\frac{2\bar{h}}{\bar{h}_n^{\,2}+\bar{\lambda}^2}(-1)^{\frac{n-1}{2}}\right]$$

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$$\overline{\varphi}_n = \left(\frac{\overline{h}_n}{\overline{\lambda}}\right)^2 (-1)^{\frac{n-1}{2}}$$

$$\alpha_2 = \frac{1}{2(1+\nu)}\left(\frac{4}{\pi}\right)^2 l_r^{-2} s_r$$

$$\eta_{l\,n} = \frac{\pi}{2}\sqrt{\frac{1-2\nu}{2(1+\nu)}}\xi_n l_r\,,\quad \eta_{t\,n} = \frac{\pi}{2}\xi_n l_r\,,$$

$$\varsigma_n = \frac{2K_1(\eta_{l\,n}) + \eta_{l\,n}K_0(\eta_{l\,n})}{2K_1(\eta_{t\,n}) + \eta_{t\,n}K_0(\eta_{t\,n})},\quad \Psi_n = K_1(\eta_{l\,n}) + \eta_{l\,n}K_0(\eta_{l\,n}) - K_1(\eta_{t\,n})\varsigma_n$$

$$\Omega_n = \left[K_1(\eta_{l\,n}) + K_1(\eta_{t\,n})\varsigma_n\right]\Psi_n^{-1}$$

$$f_n = \frac{-\alpha_2\xi_n^2\Omega_n}{n^4 - \alpha_2 d_r + \alpha_2\xi_n^2\Omega_n}\,,\quad g_n = \frac{\alpha_2\left[\Omega_n + d_r\right](4/n\pi)}{n^4 - \alpha_2 d_r + \alpha_2\xi_n^2\Omega_n}$$

$$\alpha_n = \frac{\overline{\alpha}_n}{\overline{\varphi}_n}\,,\quad \beta_n = \frac{\overline{\beta}_n}{\overline{\varphi}_n}\,,\quad \gamma_n = \frac{\overline{\gamma}_n}{\overline{\varphi}_n}\,,\quad \delta_n = \frac{\overline{\delta}_n}{\overline{\varphi}_n}$$

$$\zeta_f = -\frac{\overline{\lambda} + \sum_{n=1,3\ldots}^{\infty} f_n\,\alpha_n \overline{h}_n}{\cos\overline{\lambda}} + \frac{\overline{\lambda} + \sum_{n=1,3\ldots}^{\infty} f_n\,\gamma_n \overline{h}_n}{\cosh\overline{\lambda}}\,,\quad \zeta_g = -\sum_{n=1,3\ldots}^{\infty} g_n\,\overline{h}_n$$

$$\Gamma = \left(\overline{\lambda} + \sum_{n=1,3\ldots}^{\infty} f_n\,\alpha_n\overline{h}_n\right)\tan\overline{\lambda} + \left(\overline{\lambda} + \sum_{n=1,3\ldots}^{\infty} f_n\,\gamma_n\overline{h}_n\right)\tanh\overline{\lambda} + \frac{3\pi(1-\nu_b^2)}{8}\overline{\lambda}\,l_r r_r s_r^{-1}$$

$$A_f = \frac{\zeta_f}{\Gamma}\tan\overline{\lambda} + \frac{1}{\cos\overline{\lambda}}\,,\quad B_f = -D_f = \frac{\zeta_f}{\Gamma}$$

$$C_f = \frac{\zeta_f}{\Gamma}\tanh\overline{\lambda} - \frac{1}{\cosh\overline{\lambda}}$$

$$A_g = \frac{\zeta_g}{\Gamma}\tan\overline{\lambda}\,,\quad B_g = -D_g = \frac{\zeta_g}{\Gamma}$$

$$C_g = \frac{\zeta_g}{\Gamma}\tanh\overline{\lambda}$$

where the Poisson's ratio of the soil medium is ν; $K_m(\ )$ denotes the modified Bessel function of the 2nd kind of order *m*; and n is the mode number of the Fourier series expanded in the vertical direction.

The first and the second terms of the closed form formula in Eq. (2) are associated with the kinematic bending and the inertial bending, respectively. It should be noted that the following non-dimensional factor, the coefficient of the second term, is directly responsible for the effect of the lateral load relative to the deformation of the soil medium:

$$f_r = \frac{1}{\gamma_s} \frac{V}{E_p H^2}. \tag{3}$$

It is obvious that if the above factor f_r and the aforementioned normalized parameters, including the material damping h_g and the Poisson's ratios ν and ν_b, are compatible with arbitrary soil-pile-structure systems, the normalized bending strains become equal to each other. The factor f_r is a complex value since a phase lag generally appears between the lateral load V and the mean shear strain of the soil medium γ_s. Therefore, this factor can be written by the following formula:

$$f_r = F_r e^{i\phi_r}, \tag{4}$$

where

$$F_r = \left| \frac{1}{\gamma_s} \frac{V}{E_p H^2} \right| \text{ and}$$

$$\phi_r = \arg\left(\frac{V}{\gamma_s} \right).$$

The factor F_r can be evaluated by estimating the maximum values of the lateral load V and the mean shear strain γ_s. In this study, the lateral load V is defined as an independent parameter that represents the base shear force of arbitrary superstructure and footing. Therefore, the lateral load V is evaluated independently in this study. In general, the maximum value of the lateral load V can be approximately estimated by using response spectra (base shear forces) for ground motions. On the one hand, the maximum value of the mean shear strain γ_s can be estimated by using deformation response spectra for ground motions at the bedrock, or, on the other hand, conventional numerical techniques such as SHAKE [Schnabel et al. (1972)] may be used. From the viewpoint of engineering practice, the factor F_r should be less than about 10^{-3}.

The determination of the factor ϕ_r is difficult because there have been few investigations into the phase lag between the lateral load V and the mean shear strain γ_s (ground motion), especially in soil-pile-structure systems where the kinematic interaction dominates. Murono and Nishimura (2000) show fundamental characteristics of the phase lag between the lateral load and the ground displacement. The results of their study indicate that three types of phase lag predominantly occur. These are associated with the ratio of the natural period of a soil-pile-structure system T_s and the fundamental natural period of a soil medium T_g in a soil-pile-structure system where the kinematic interaction dominates, and are as follows: (1) In the case of $T_s/T_g \leq 1$, the phase lag of the lateral load with respect to the ground motion tends to become zero; (2) in the case of $T_s/T_g \approx 1$, the phase lag tends to become $-\pi/2$; and (3) in the case of $T_s/T_g \geq 1$, the phase lag tends to becomes $-\pi$ (in practice, $-3\pi/4$ would be more appropriate, in accordance with the design coefficients described in their study). These characteristics can clearly be seen in the case where the input motion for the systems is assumed to be a harmonic wave whose fundamental period is identical to the fundamental natural period of the soil medium. Moreover, it should be noted that similar characteristics can also be seen in the case where earthquake waves are applied to the systems. This implies that the fundamental natural period of the soil medium dominates when these systems are subjected to earthquake waves. Therefore, the specific period focused on in this study is considered to be appropriate. Luo et al. (2002) verified the validity of the above characteristics through the application of the seismic deformation method (SDM) to a simulative analysis for pile foundations embedded in soft soil that experienced the Hyogoken-Nanbu earthquake. Accordingly, this study follows the phase-lag characteristics described above.

NORMALIZED BENDING STRAIN AND OPTIMAL PILE RADIUS

Fig. 2 shows the variations in normalized bending strains as functions of the slenderness ratio a/H with different values of the factor F_r for the phase lag $\phi_r = 0$. The absolute values of the normalized bending strains and related terms evaluated by Eq. 2 are shown. It is assumed in Fig. 2 that the stiffness ratio $E_g/E_p = 0.001$; the stiffness ratio $E_g/E_b = 0.05$; the mass density ratio $\rho_p/\rho_g = 1.25$; the material damping $h_g = 0.05$; and the Poisson's ratios $\nu = \nu_b = 0.45$. Fig. 2(a) indicates that the normalized bending strain due to the inertial bending, which is identical to the second term of Eq. 2, significantly decreases as a/H increases. When the factor F_r increases, the strain gradually increases within the range of a/H shown. Fig. 2(b) shows the variation in normalized bending strain due to the kinematic bending, which is identical to the first term of Eq. 2. There can be no variation of F_r since the bending strains are essentially independent of F_r, as shown in Eq. 2. Therefore, a single line is shown in Fig. 2(b). The bending strains become zero when the slenderness ratio a/H approaches zero. The bending strains increase almost linearly up to a local maximum ($a/H \approx 0.1$), and gradually decrease beyond the local maximum. This indicates the presence of a worst-case slenderness ratio that maximizes the kinematic bending strain. From the viewpoint of engineering practice, typical slenderness ratios a/H may range approximately from 0.01 to 0.1. It is conceivable, therefore, that the normalized bending strain due to the kinematic bending may increase almost linearly as the slenderness ratio a/H increases. It is noted that an opposite change in the bending strains with a/H, due to the inertial bending and the kinematic bending, occurs within the practical range of a/H.

Figs. 2(c) and 2(d) indicate the presence of a local minimum area where the normalized bending strains due to both the inertial bending and the kinematic bending are minimized. In this case, the local minimum area occurs for small F_r (i.e., $\leq 5.0\times10^{-5}$) within the practical range of a/H. This implies the presence of a slenderness ratio a/H (that is, a radius) that can appropriately minimize the bending strains. The presence of this a/H is apparently attributed to the opposite change in the inertial and kinematic bending strains with a/H. It should be noted that the presence of the local minimum may largely depend on the value of F_r.

Fig. 3 shows the variations in normalized bending strains with different values of the phase lag ϕ_r for the factor $F_r = 5.0\times10^{-5}$. Assumptions identical to those in Fig. 2 are made in Fig. 3 for the non-dimensional parameters E_g/E_p, E_g/E_b, ρ_p/ρ_g, the material damping h_g, and the Poisson's ratios ν and ν_b. A single line is shown in each of Figs. 3(a) and 3(b) since each graph shows the absolute values of the normalized bending strains, which are independent of the phase lag ϕ_r. Fig. 3(c) indicates that the normalized bending strains at the local minimum ($a/H \approx 0.05$) gradually decrease as ϕ_r decreases, while the slenderness ratio a/H minimizing the normalized strains do not substantially change with changes in ϕ_r. Fig. 3 also indicates that when ϕ_r becomes $-\pi$, the normalized bending strains converge to zero. The reason for this is that the normalized bending strain due to the inertial bending becomes equal to that due to the kinematic bending, but in exactly the opposite direction. From a practical point of view, however, this may not be realistic because the phase lag ϕ_r probably converges to about $-3\pi/4$ in the case where $T_s/T_g \geq 1$, as described by Murono and Nishimura (2000).

Fig. 4 shows the effects of the stiffness ratio E_g/E_p upon the normalized bending strains for the factor $F_r = 5.0\times10^{-5}$ and the phase lag $\phi_r = 0$. The same assumptions in Fig. 2 and 3 are applied for the non-dimensional parameters in Fig. 4. Figs. 4(a) and 4(b) reveal that the normalized bending strain due to the inertial bending decreases over the entire range of a/H as the stiffness ratio E_g/E_p increases. In contrast, as E_g/E_p increases, the normalized bending strain due to the kinematic bending increases rapidly as the slenderness ratio a/H locally maximizing the normalized bending strain increases. As a result, a local minimum area is generated for large E_g/E_p (e.g., $\geq 10^{-3}$), as shown in Figs. 4(c) and 4(d).

Fig. 5 shows the variations in normalized bending strains with various values of the non-dimensional parameters E_g/E_b, h_g, ρ_p/ρ_g, and ν for the factor $F_r = 5.0\times10^{-5}$, the phase lag $\phi_r = 0$, and $\nu_b = 0.45$. Figs. 5(a), (b), (c) and (d) indicate that they have little effect upon the normalized bending strains. Especially for E_g/E_b, this result means that differences in the degree of restraint of the rotational movement at the toe of the piles are almost negligible within the range of E_g/E_b shown in Fig. 5(a).

CRITERIA FOR DETERMINING THE OPTIMAL PILE RADIUS

Figs. 6 and 7 show criteria for easily estimating the slenderness ratio a/H that locally minimizes the normalized bending strains in soil-pile-structure systems where the kinematic interaction dominates. These criteria are derived from Eq. 2 for $E_g/E_b = 0.05$, $h_g = 0.05$, $\rho_p/\rho_g = 1.25$, and $\nu = \nu_b = 0.45$. These figures indicate that once the values of E_g/E_p and F_r, as well as the phase lag ϕ_r, are determined, the slenderness ratio a/H can be evaluated from curves indicating the slenderness ratios a/H for various combinations of E_g/E_p and F_r. It is apparent that the optimal pile radius is equal to the product of the corresponding a/H and the length of the pile H. In addition, these figures show the slenderness ratios a/H that locally maximize the normalized bending strains for combinations of E_g/E_p and F_r. In these figures, a factor P_r is also presented as a ratio (expressed as a percentage) of the normalized bending strain at the local minimum to that at the local maximum, as shown in Fig. 6(b). If the factor P_r is one-hundred percent, the normalized bending strains at the local minimum and the local maximum are identical to each other, and usually the local minimum and the local maximum almost disappear. Therefore, as shown in Fig. 6(a) and Fig. 7, no slenderness ratio a/H minimizing or maximizing the normalized bending strains appears in the region to the right of the P_r =100% curve.

As a whole, it appears that the slenderness ratios a/H at local minima increase as F_r increases, while the ratios a/H decrease as E_g/E_p increases. Moreover, it is noted that the factor P_r becomes small as E_g/E_p increases or as F_r decreases. This means that a substantial decrease in the normalized bending strains at the local minimum would be expected within a range of large E_g/E_p and small F_r. In addition, this decrease seems to be more significant for small ϕ_r than for large ϕ_r.

Herein, we consider a typical soil-pile-structure system. The following properties of the system are considered: $H = 20$ m, $E_p = 2.5\times10^7$ kN/m^2, $E_g = 5.8\times10^4$ kN/m^2, $\gamma = 5.0\times10^{-3}$, and $V = 400.0$ kN. In case of a pile group, for instance, the lateral force V can be approximated as the average lateral force acting on the piles. It is expected, however, that the precision of the optimal pile radius evaluated by this criteria will decrease to some extent if pile-to-pile interactions dominate among the piles. In this system, the phase lag ϕ_r is assumed to be $-\pi/2$. It is also assumed that there are no significant differences in the other non-dimensional parameters. The values of $\log F_r$ and $\log E_g/E_p$ can be calculated as -5.10 and -2.63, respectively. Therefore, the following values are obtained from Fig. 7(a): the slenderness ratio a/H at the local minimum of the normalized bending strains is about 0.023; the slenderness ratio a/H at the local maximum is about 0.115; and P_r is about 34%. Accordingly, the radius of the pile minimizing the bending strains at the head of the pile is about 0.46 m (0.023×20 m). The variation of normalized bending strains with a/H from the local minimum and the local maximum can be approximated as a straight line. Therefore, an approximate value of the ratio of the normalized bending strain at an arbitrary a/H to that at the local minimum can be obtained by the following formula.

$$D_r = \frac{P_r}{\dfrac{100-P_r}{a/H_{(Max.)} - a/H_{(Min.)}}\left(a/H - a/H_{(Max.)}\right)+100} \tag{5}$$

If it is assumed that the radius a of the pile is, for instance, 0.8 m (the conventionally used value), the value of the ratio D_r can be estimated to be about 0.74. This implies that a $26\,\%$ decrease in the bending strain would be expected if the radius a were changed from 0.8 m to 0.46 m. In practical applications, therefore, Figs. 6 and 7, and also Eq. 5, may be useful for estimating the slenderness ratio a/H (radius a) that minimizes the bending strains at the head of the piles, and for evaluating the effect of the slenderness ratio a/H upon the bending strains at arbitrary a/H using the factor D_r.

CONCLUSIONS

In the present study, the following may be concluded:

1. In order to evaluate the characteristics of the bending strains of fixed-head piles embedded in a homogeneous soil in soil-pile-structure systems where the kinematic interaction dominates, closed form formulae are derived using three-dimensional wave propagation theory. A general expression for the closed form formulae can be obtained by normalizing the bending strain with respect to a mean shear strain of a soil stratum γ_s. It is found that the normalized bending strains can be expressed by the slenderness ratio a/H, the ratio of soil and pile stiffness E_g/E_p, a factor F_r, and a phase lag ϕ_r, which represent dynamic characteristics of loading at the head of the piles and deformation of the soil.

2. The normalized bending strain due to the inertial bending decreases rapidly as the slenderness ratio a/H increases. On the one hand, the normalized bending strain due to the kinematic bending becomes zero when a/H is zero, whereas the normalized bending strain has a local maximum in a higher a/H region and gradually decreases beyond the local maximum. The variation of normalized bending strains with a/H indicates that a slenderness ratio a/H (radius) that minimizes the normalized bending strains at the head of the piles may appear, depending on the values of E_g/E_p, F_r, and ϕ_r. In addition, a local maximum of the normalized bending strain mainly due to the kinematic bending may also appear in the higher a/H region. Parametric studies indicate that the local minimum is more easily generated as F_r becomes smaller or as E_g/E_p becomes larger. Moreover, the normalized bending strains at the local minimum gradually decreases as the phase lag ϕ_r decreases.

3. Criteria are provided for easily estimating the pile radius that minimizes the normalized bending strains. It is noted that the factor P_r, the ratio of the normalized bending strain at the local

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minimum and that at the local maximum, becomes small as E_g/E_p increases or as F_r decreases.

From these criteria, effects of the slenderness ratio a/H at the local minimum upon the normalized bending strains at arbitrary values of a/H can also be estimated using the factor D_r.

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APPENDIX A. NOTATION

The following symbols are used in this paper:

a = radius of pile;

$a/H_{(Max.)}$ = slenderness ratio of pile at local maximum of bending strain;

$a/H_{(Min.)}$ = slenderness ratio of pile at local minimum of bending strain;

E_b = Young's modulus of bedrock;

E_g = Young's modulus of soil;

E_p = Young's modulus of pile;

h_g = damping constant of soil;

H = length of pile;

I = geometrical moment of inertia of pile;

$i = \sqrt{-1}$ = imaginary unit;

K_m = modified Bessel function of the 2nd kind of order m;

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K_r	=	rotational stiffness at the toe of pile;
n	=	mode number of the Fourier series expanded in the vertical direction;
t	=	time;
u_g	=	amplitude of horizontal excitation;
V	=	lateral load acting on the top of pile;
V_{sb}	=	shear velocity of bedrock;
γ_s	=	mean shear strain of soil stratum;
ν	=	Poisson's ratio of soil;
ν_b	=	Poisson's ratio of bedrock;
ρ_g	=	mass density of soil;
ρ_p	=	mass density of pile;
ω	=	circular frequency.

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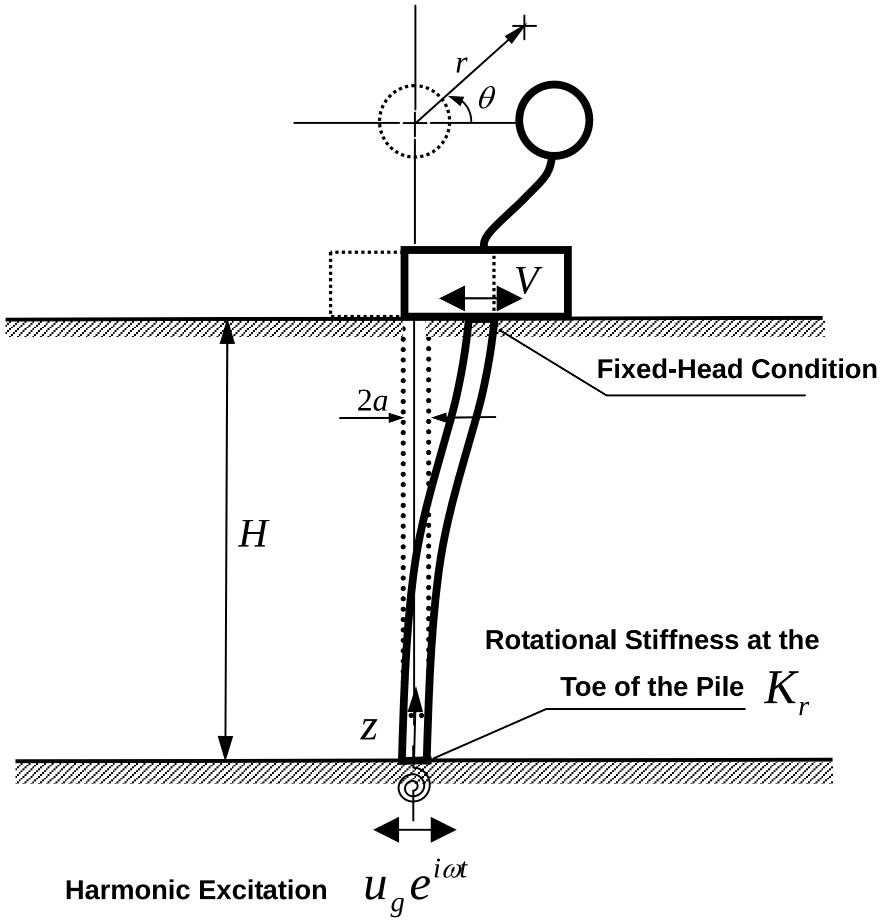

Fig.1 Analytical soil-pile-structure model for a homogeneous soil medium and a fixed- head pile supported by rotationally compliant bedrock: the system is excited by vertically-propagating S-waves

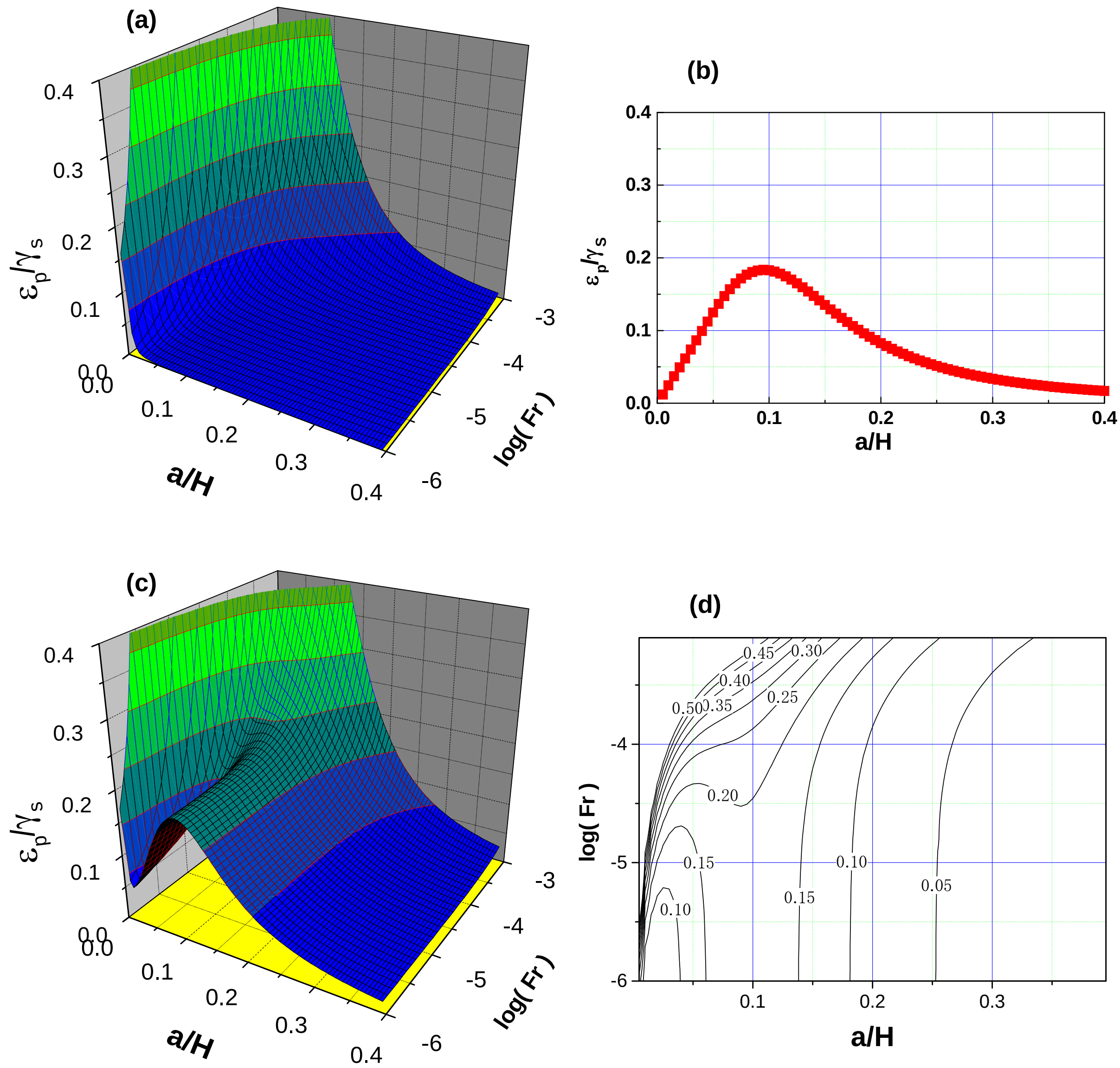

Fig.2 Variation of normalized bending strains with F_r [(a) inertial bending (b) kinematic bending (c) total bending (d) contours of normalized bending strains ε_p/γ_s]: $\phi_r = 0$ and $E_g/E_p = 0.001$

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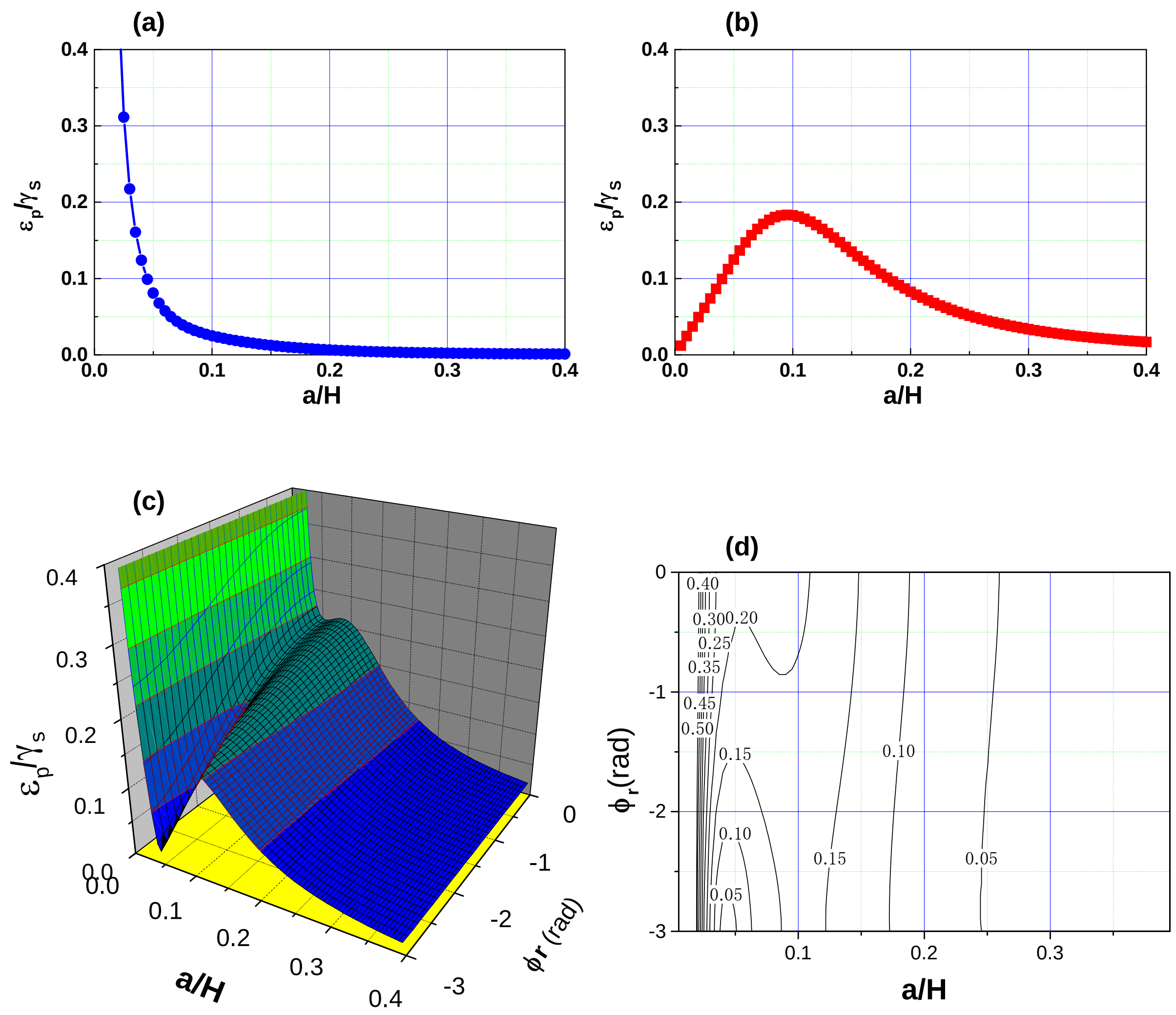

Fig.3 Variation of normalized bending strains with ϕ_r [(a) inertial bending (b) kinematic bending (c) total bending (d) contours of normalized bending strains ε_p/γ_s]: $F_r = 5.0\times10^{-5}$ and $E_g/E_p = 0.001$

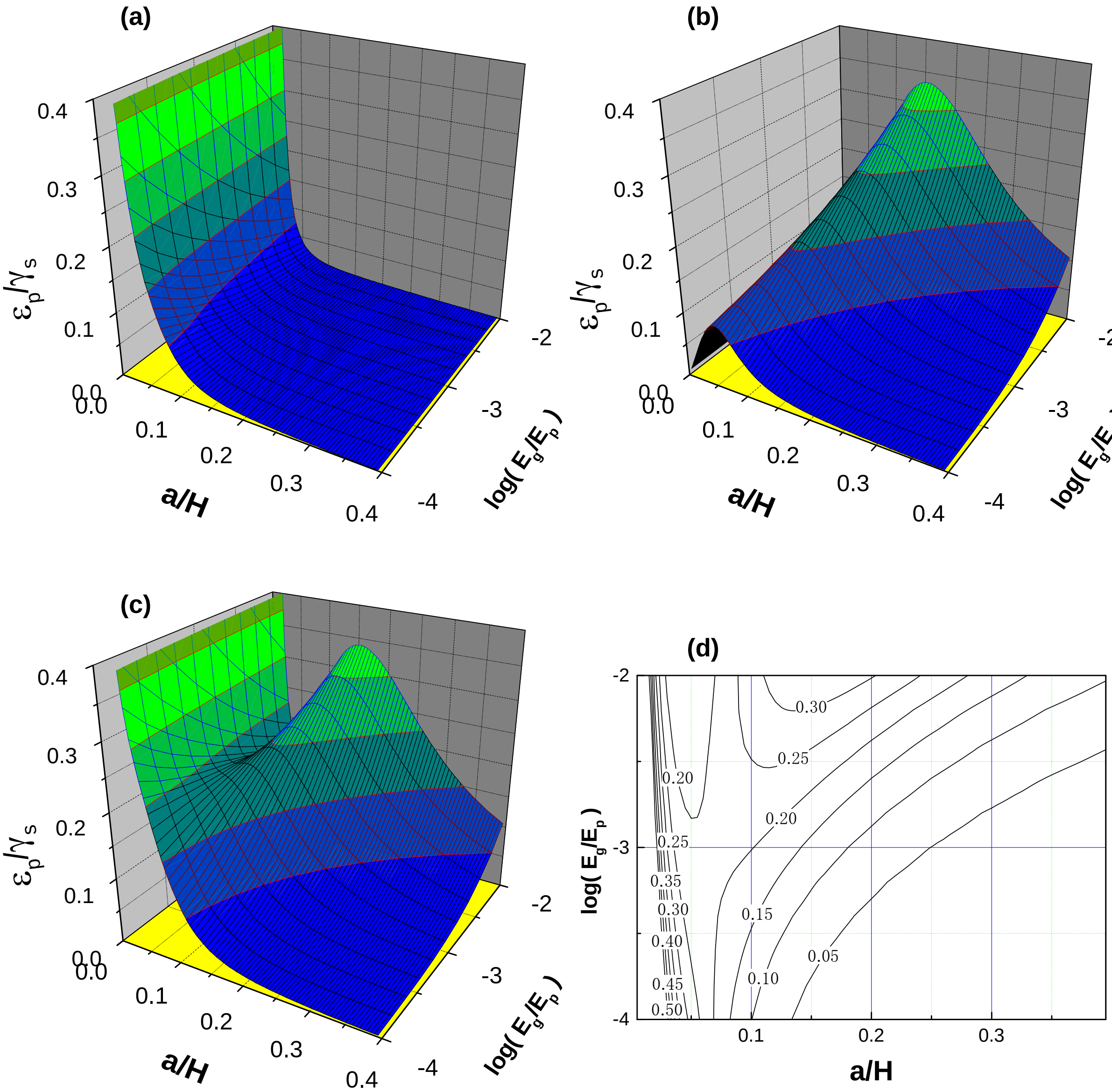

Fig.4 Variation of normalized bending strains with E_g/E_p [(a) inertial bending (b) kinematic bending (c) total bending (d) contours of normalized bending strains ε_p/γ_s]: $F_r = 5.0\times10^{-5}$ and $\phi_r = 0$

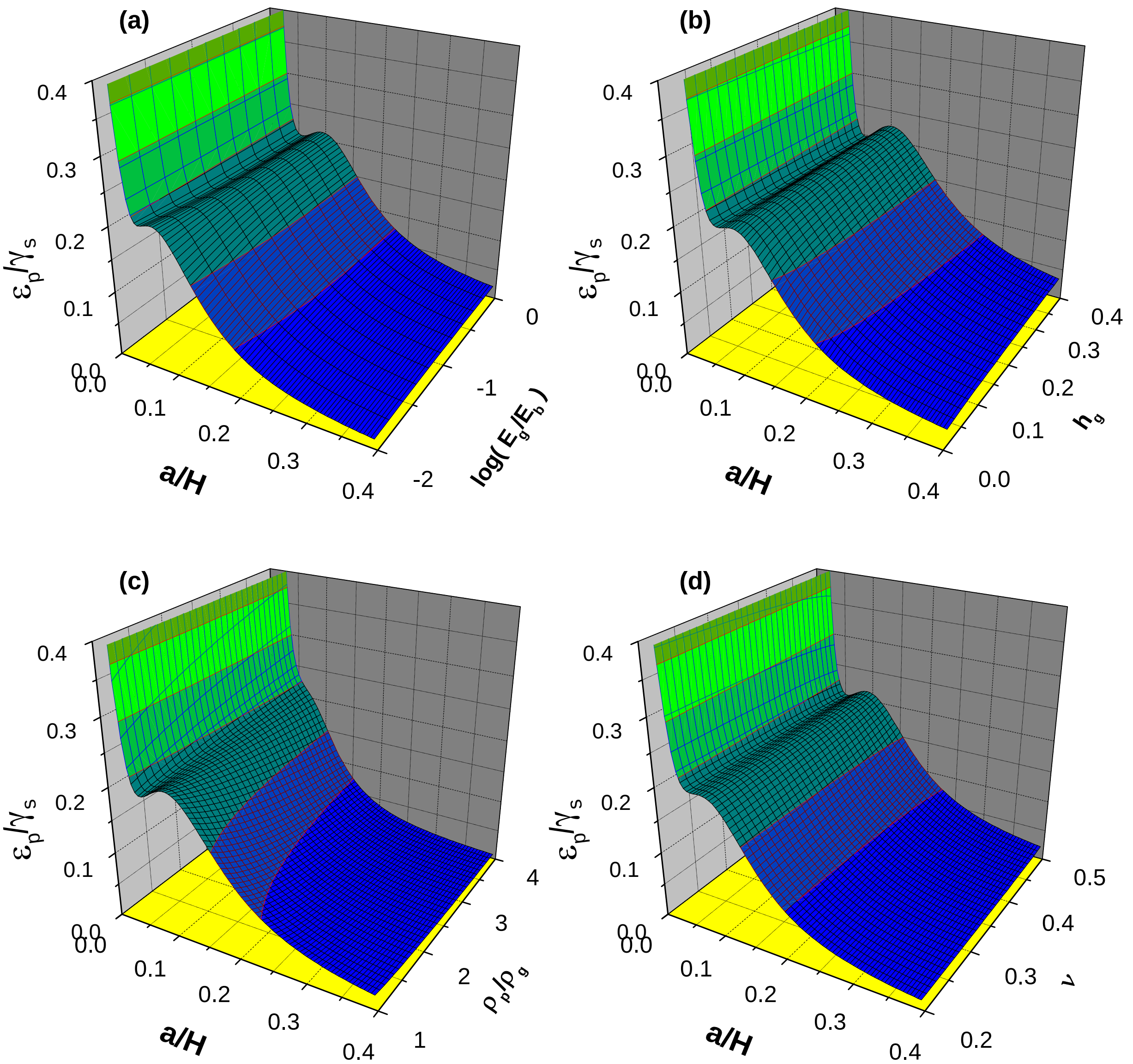

Fig.5 Variation of normalized bending strains with various parameters [(a) E_g/E_b (b) h_g (c) ρ_p/ρ_g (d) ν]: $F_r = 5.0\times10^{-5}$ and $\phi_r = 0$

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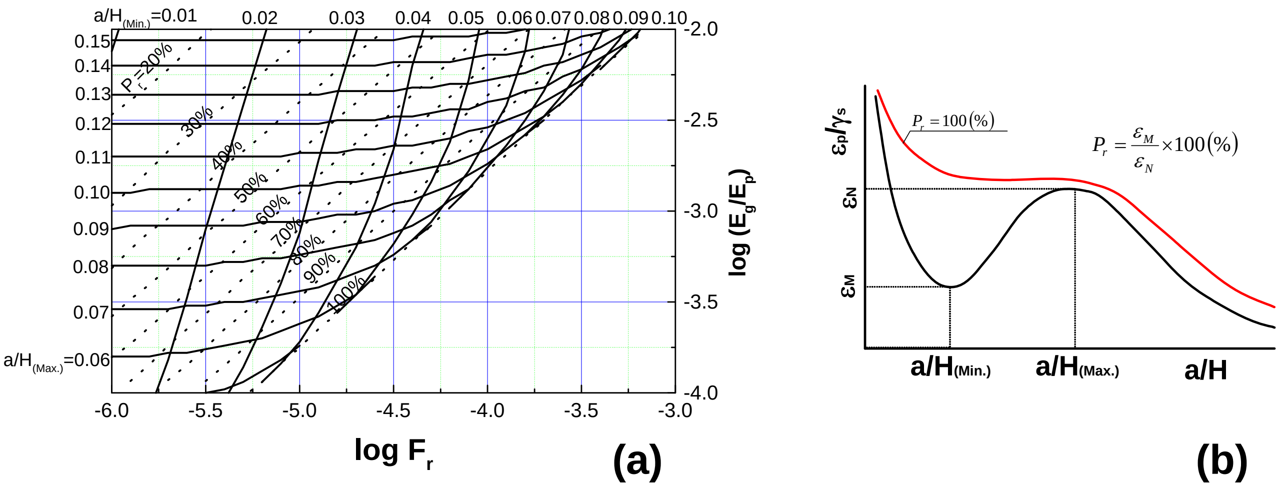

Fig.6 Criteria for evaluating slenderness ratios a/H that minimizes normalized bending strains at the head of a fixed-head pile with E_g/E_p and F_r when kinematic interaction dominates in soil-pile- structure systems [(a) criteria for $T_s/T_g \leq 1$ ($\phi_r = 0$) (b) definition of corresponding factors in a/H - ε_p/γ_s relations]

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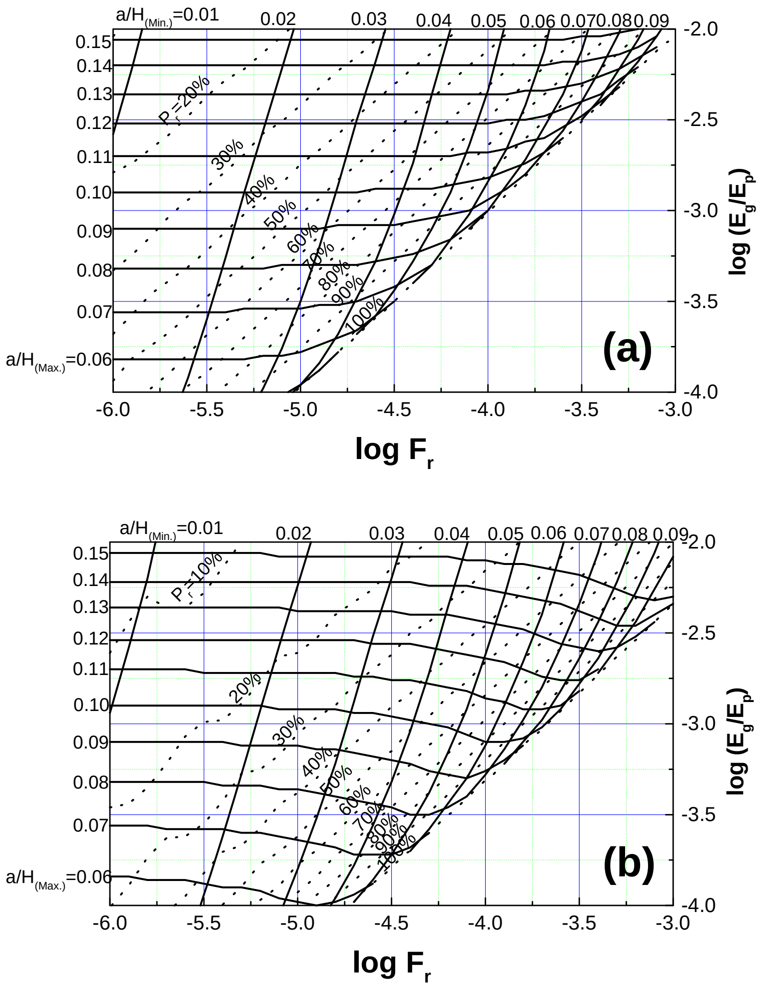

Fig.7 Criteria for evaluating slenderness ratios a/H that minimizes normalized bending strains at the head of a fixed-head pile with E_g/E_p and F_r when kinematic interaction dominates in soil-pile- structure systems [(a) for $T_s/T_g \approx 1(\phi_r = -\pi/2)$ (b)for $T_s/T_g \geq 1(\phi_r = -3\pi/4)$]